FORECASTING AFTERNOON MIXING DEPTHS AND TRANSPORT WIND SPEEDS

MARVIN E. MILLER

Air Resources Field Research Office,[1] ESSA, Cincinnati, Ohio

ABSTRACT

A method is presented to estimate the vertical extent of atmospheric mixing during the afternoon and the average transport wind speed, i.e., the average wind speed within the mixing layer. Afternoon mixing depth is assumed to be dependent upon the observed difference between the maximum afternoon surface temperature and the mean virtual temperature of the standard atmospheric layer containing the top of the mixing depth. Transport wind speed is assumed to be a function of the wind speed at the level nearest the center of the mixing depth. Based on special forecast material prepared by the National Meteorological Center, procedures are outlined for forecasting afternoon mixing depths and transport wind speeds.

1. INTRODUCTION

The two most important meteorological variables that determine the dilution of air pollutants over urban areas are (1) the vertical depth through which the dispersion takes place (i.e., the "mixing depth"), and (2) the mean transport wind speed in that layer. An estimate of this afternoon's mixing depth is obtained by finding the height above the surface of the dry adiabatic intersection of the day's observed maximum surface temperature with the day's 1200 GMT observed vertical temperature profile. Holzworth [2] used mean radiosonde observations and normal maximum surface temperatures to estimate monthly mean afternoon mixing depths for 45 stations in the conterminous United States. In the National Air Pollution Potential Forecast Program (Miller and Niemeyer [3]), it is of special interest to anticipate the magnitude of mixing depths and transport wind speeds during the afternoon. At that time they normally reach their maximum values and hence represent the *best* dilution conditions that will occur during the entire day. When referred to established criteria, their values can then serve as important indices or guides in the decision-making process that may lead to the issuance of an air pollution potential advisory. This report presents an objective method for forecasting the afternoon mixing depths and associated mean transport wind speeds.

2. AFTERNOON MIXING DEPTHS

DEVELOPMENT

Estimates of afternoon mixing depth are a function of the 1200 GMT vertical temperature profile and the maximum afternoon temperature, but, in view of the inherent difficulty in forecasting the *detail* of vertical temperature profiles, it was decided to base mixing depth forecasts on relationships between less direct but more readily forecastable parameters. An obvious relationship is that between the mixing depth and the potential temperature difference between the surface maximum temperature and the temperature at some constant pressure surface aloft.

To test this relationship, surface and upper-air observations from Pittsburgh, Pa., were utilized; the 850-mb. level was arbitrarily specified as the constant pressure surface aloft. When mixing depths were plotted against the differences between the afternoon maximum surface potential temperature, θ_{sfc}, and the 850-mb. potential temperature, θ_{850} (at 1200 GMT), the scatter expanded away from the point where $\theta_{sfc}-\theta_{850}=0$. This variation was essentially due to the occurrence of a wide range of lapse rates in the layer between the top of the mixing depth and the 850-mb. level. Thus, the more the top of the mixing depth departed from the 850-mb. level, the wider was the range of mixing depth values for $\theta_{sfc}-\theta_{850}$. In addition, however, even in cases when $\theta_{sfc}-\theta_{850}=0$, i.e., when the mixing depth coincided with the 850-mb. level, some variations in mixing depths occurred because of the daily and seasonal variations of 850-mb. heights. Monthly mean 850-mb. heights, based on 10 yr. of data, have been given by Ratner [4]; the maximum difference between any two months at Pittsburgh was 101 m.

Since in the relationship between $\theta_{sfc}-\theta_{850}$ and the mixing depth, minimum scatter was observed about the point where $\theta_{sfc}-\theta_{850}=0$, it was hypothesized that the scatter of all points would be minimized if the mixing depths were separated into smaller layers and related to the difference between the temperature of the surface (t_{sfc}) at the time of maximum, and the 1200 GMT mean

[1] Robert A. Taft Sanitary Engineering Center, Division of Air Pollution, Public Health Service, U.S. Department of Health, Education, and Welfare. Author's present affiliation: ESSA Weather Bureau State Climatologist, Columbus, Ohio.

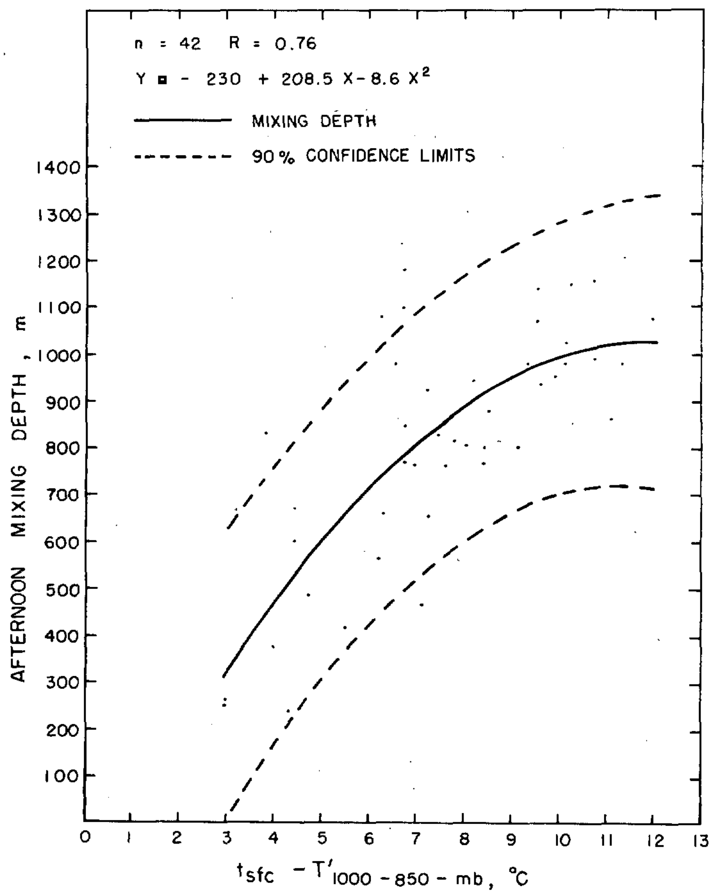

FIGURE 1.—Afternoon mixing depths whose upper limits were below the 850-mb. level, as a function of $(t_{sfc}-T'_{1000-850\ \mathrm{mb.}})$ at Dayton, Ohio.

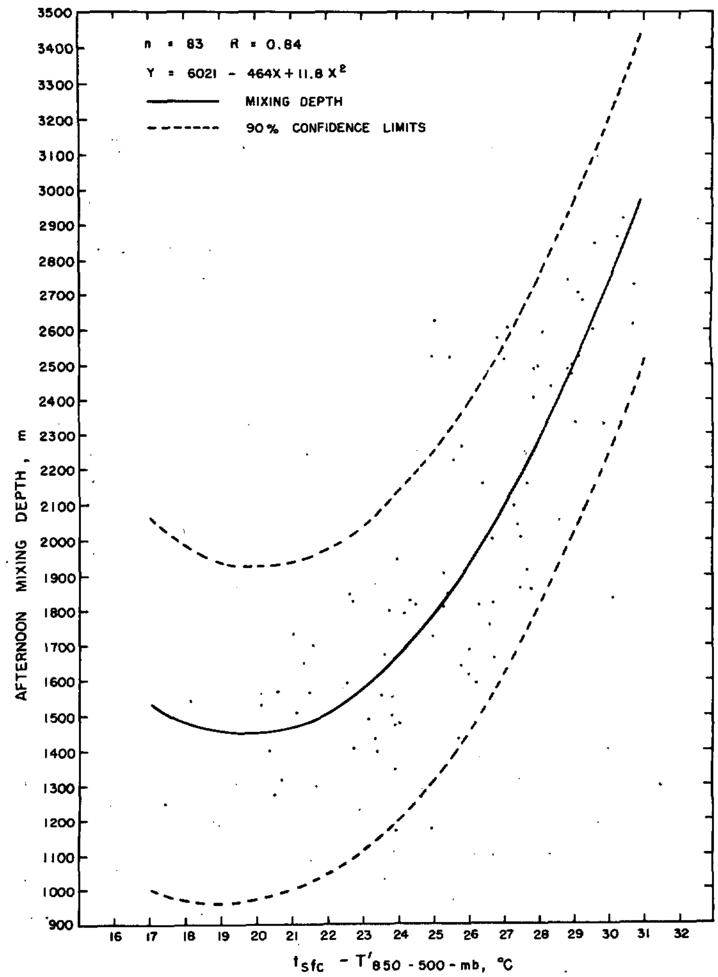

FIGURE 2.—Afternoon mixing depths whose upper limits were within the 850–500-mb. layer, as a function of $(t_{sfc}-T'_{850-500\ \mathrm{mb.}})$ at Dayton, Ohio.

virtual temperature, T', of the layer (1000–850 mb., 850–500 mb., etc.) that included the top of the mixing depth. These layers were selected to coincide with the basic levels used by the National Meteorological Center (NMC) in their prognostic models. Further subdivision of the layers was not recommended, since all other levels are linear interpolations of the basic levels. A computer program was written to obtain statistical information on the relationships between mixing depth and temperature difference $(t_{sfc}-T')$ at 67 rawinsonde stations within the conterminous United States. Input data from daily rawinsonde runs during calendar year 1964 were used to determine coefficients for several types of regression equations; data for days with more than a trace of precipitation were excluded from the computations because of differences in the physical processes experienced by rising saturated air parcels as compared with rising dry parcels.

The best least squares fit of the data was obtained with parabolic regression equations. Table 1 summarizes the resulting correlation coefficients for the mixing depths as

TABLE 1.—*Summary of correlation coefficients for the parabolic relationship between afternoon mixing depths, whose upper limits were below the 850-mb. level, and* $(t_{sfc}-T'_{1000-850\ mb})$.

Index of correlation	<.56	.56–.60	.61–.65	.66–.70	.71–.75	.76–.80	.81–.85	>.85
Number of stations	2	1	0	4	8	12	14	9

TABLE 2.—*Summary of standard errors of mixing depths for the parabolic relationship between afternoon mixing depths, whose upper limits were below the 850-mb. level and* $(t_{sfc}-T'_{1000-850\ mb})$.

Standard error (m)	120–139	140–159	160–179	180–199	200–219	220–239
Number of stations	1	8	17	18	5	1

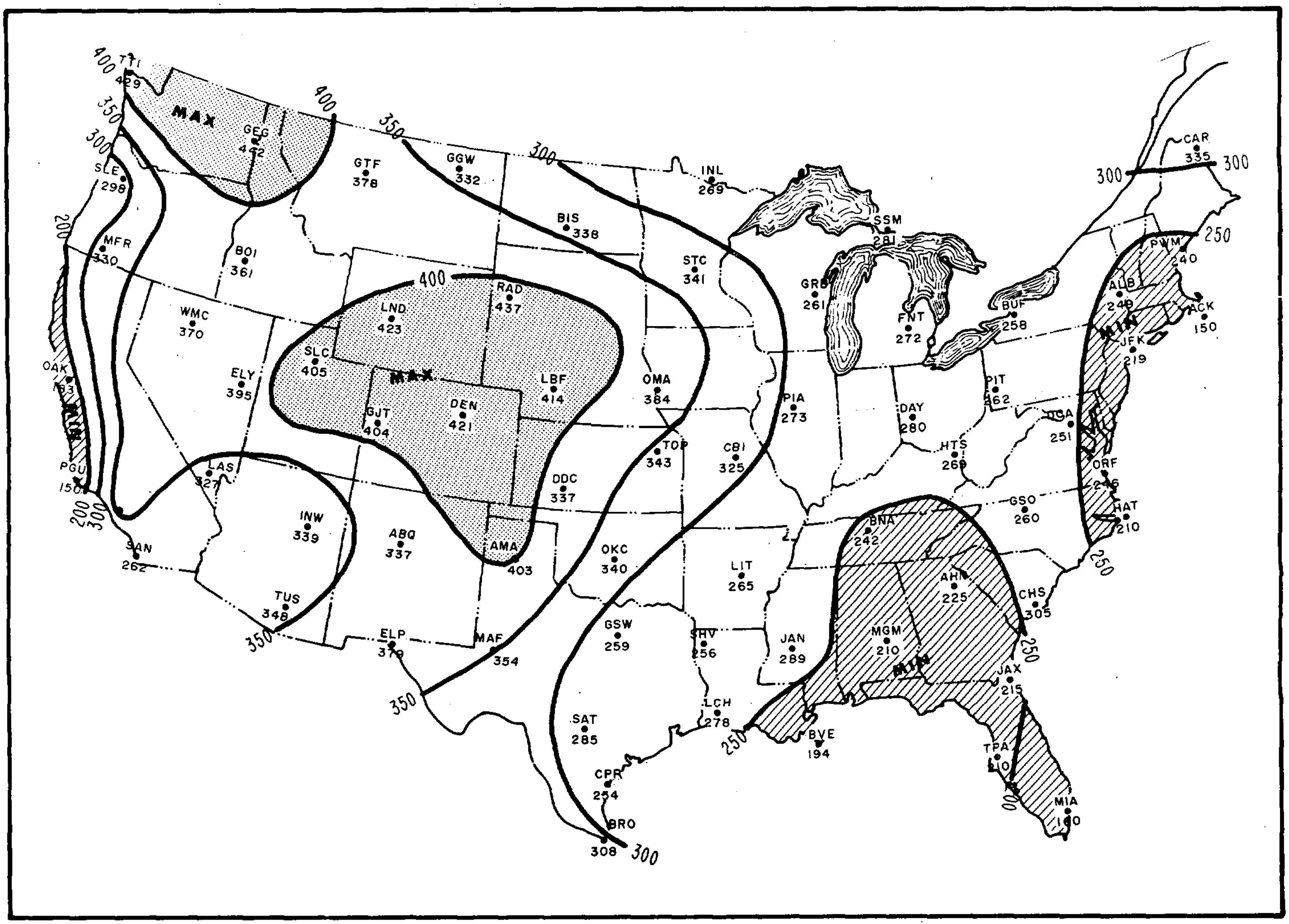

FIGURE 3.—Isopleths of standard errors of mixing depth (meters) for the relationship between afternoon mixing depth within the 850–500-mb. layer and $(t_{sfc}-T'_{850-500\text{ mb.}})$.

parabolic functions of $(t_{sfc}-T')$ when the tops of the mixing layers were below the 850-mb. level. Table 2 summarizes the standard errors of mixing depths about these equations. The standard errors of mixing depths were less than 200 m. at 44 of the 50 stations for which comparisons were made. Stations with elevation 750 m. or higher were excluded from these summaries because of their nearness to the 850-mb. level.

Figures 1 and 2 show the individual observations used in deriving the statistical equations and the 90 percent confidence limits for the relationship between mixing depth and $t_{sfc}-T'$ when the top of the mixing depth fell respectively within the 1000–850-mb. and 850–500-mb. layers at Dayton, Ohio. Similar information was obtained for all stations shown in figure 3.

Table 3 summarizes the computed correlation coefficients for the parabolic relationship between afternoon mixing depths and $t_{sfc}-T'$ when the tops of the mixing depths fell between 850 and 500 mb. Figure 3 shows the geographical distribution of the calculated standard errors of mixing depths when the tops of the mixing layers fell within the 850–500-mb. layer.

Figure 4 shows the confidence limits (plus or minus) within which a mixing depth of 2000 m. occurred 90 percent of the time during 1964 over the conterminous United States. Maximum error was observed at those locations with higher station elevations.

TABLE 3.—*Summary of the correlation coefficients for the parabolic relationship between afternoon mixing depths, whose upper limits were between 850 and 500 mb., and* $(t_{sfc}-T'_{850-500\text{ mb.}})$.

Index of correlation	<.56	.56–.60	.61–.65	.66–.70	.71–.75	.76–.80	.81–.85	.86–.90	>.90
Number of stations	3	1	2	2	5	12	11	15	16

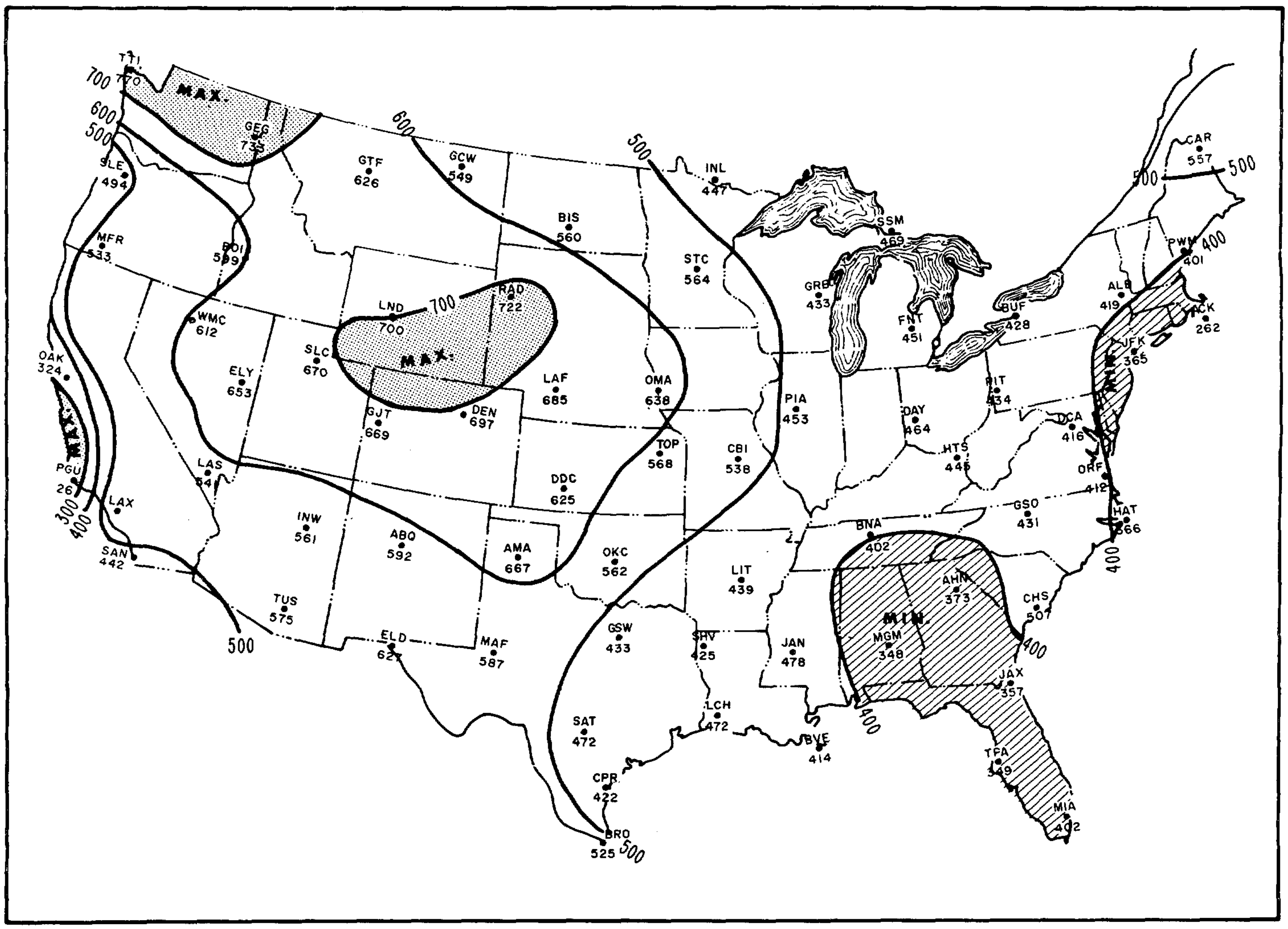

FIGURE 4.—Isopleths of 90 percent confidence limits (meters) about 2000-m. mixing depths.

FORECASTING PROCEDURE

To estimate tomorrow afternoon's mixing depth [2] at some location, NMC first makes a forecast of tomorrow afternoon's maximum surface potential temperature, θ_{sfc}, based on forecasts of tomorrow's 1200 GMT station pressure and maximum afternoon surface temperature.

Indirect 24-hr. temperature forecasts for the 850- and 500-mb. levels are then obtained from this morning's numerical weather prediction forecasts. The computer converts these temperatures to potential temperatures. If $\theta_{850}-\theta_{sfc}$ is positive, the top of the mixing depth is expected to occur below 850 mb. If, however, $\theta_{850}-\theta_{sfc}$ is negative, the top of the mixing depth is expected to occur above 850 mb., and the computer proceeds to calculate $\theta_{500}-\theta_{sfc}$ and to test its sign. Similarly, if this difference is positive, the top of the mixing depth is expected to occur in the layer 850–500 mb. and if negative, the top of the mixing depth is forecast to extend above 500 mb.

After finding the standard layer that includes the top of the mixing depth, the computer calculates the forecast thickness and then the mean virtual temperature of this layer for 1200 GMT tomorrow. Once the independent variable, $t_{sfc}-T'$, is known, a forecast of tomorrow afternoon's mixing depth is obtained from the parabolic regression equation appropriate for the layer that contains the top of the mixing layer. For purposes of the air pollution potential forecast program it is unnecessary to know the value of the mixing depth when it extends above 500 mb. In such cases, the mixing depth forecast indicates only that the depth will be deeper than the height of the 500-mb. surface above the ground.

[2] An estimate of this afternoon's mixing depth is obtained by finding the height above the surface of the dry adiabatic intersection of today's forecast maximum surface temperature with today's 1200 GMT observed vertical temperature profile.

3. TRANSPORT WIND SPEEDS

DEVELOPMENT

Since wind speed normally varies to some extent with height, the average wind speed through the mixing depth was chosen as a convenient representation of the horizontal transport of air within the mixing layer. At most Weather Bureau rawinsonde stations, winds aloft are not observed during the normal diurnal time of maximum atmospheric mixing, i.e., mid-afternoon. Observations from Salt Lake City, Pittsburgh, and Nashville were used to determine the observation time best describing the afternoon transport wind. For *each* of the observation times, wind speeds were averaged through the *afternoon* mixing depth. These mean wind speeds were correlated with the observed daily average afternoon (1200–1600 LST) *surface* wind speeds. Table 4 summarizes these results. Winds aloft data at 0000 GMT were near the usual times of maximum afternoon heating and gave the highest correlation coefficients. Therefore, wind information at 0000 GMT was assumed to give the best estimates of afternoon average wind speed through the mixing depths within the conterminous United States.

In a search for a suitable relationship between the transport wind speed and a forecastable parameter, three different variables were related to the 0000 GMT average wind speed through the afternoon mixing depth. These variables were the 1200 GMT transport wind speed, the 0000 GMT 850-mb. wind speed, and the 0000 GMT wind speed at the level nearest the center of the afternoon mixing layer. Table 5 shows the results of these correlations for the test locations. Clearly, the best correlations were attained with the 0000 GMT wind speeds at the level nearest the center of the mixing depth.

For each station shown in figure 5, parabolic regression equations were calculated from 1964 data for the relationship between the wind speed at the level nearest the center of the afternoon mixing depth and the average wind speed through this layer. Table 6 summarizes the correlation coefficients for this relationship. Sixty of the 67 correlations were greater than 0.85. Figure 5 shows the geographical distribution of the standard errors of mean transport wind speeds when the wind speed nearest the center of the mixing layer was used as the independent variable.

FORECASTING PROCEDURE

Forecasts of mean transport wind speeds are obtained from the parabolic relationship between the average wind speed through the mixing layer and the wind speed nearest the center of the mixing layer. Estimates of wind direction and speed for any geographical point in the United States and at any height are available through linear interpolation of the FD winds aloft forecasts (Badner and Kulawiec [1]) prepared by NMC. Hence, estimates of wind speeds nearest the center of the mixing depths are obtained directly from the FD forecasts.

TABLE 4.—*Correlation coefficients for the relationship between average transport wind speed through the afternoon mixing depth and the average afternoon surface wind speed.*

City	Winds aloft observation time (GMT)		
	1200	1800	0000
Salt Lake City, Utah	0.68	*0.77	0.83
Pittsburgh, Pa.	0.74	----------	0.85
Nashville, Tenn.	0.67	----------	0.82

*Salt Lake City is one of the few rawinsonde stations within the United States for which atmospheric soundings are available every 6 hours.

TABLE 5.—*Correlation coefficients for relationships between various wind speed parameters and the 0000 GMT average transport wind speed.*

Wind speed parameter	Salt Lake City, Utah	Nashville, Tenn.	Pittsburgh, Pa.
1200 GMT mean wind speed through the afternoon mixing depth	0.74	0.65	0.82
0000 GMT 850-mb. wind speed	0.90	0.82	0.86
0000 GMT wind speed nearest the center of the afternoon mixing depth	0.92	0.95	0.96

TABLE 6.—*Summary of the correlation coefficients for the derived parabolic relationship between the 0000 GMT wind speed nearest the center of the afternoon mixing depth and the 0000 GMT average transport wind speed.*

Index of correlation	.65–.70	.71–.75	.76–.80	.81–.85	.86–.90	.91–.95	>.95
Number of Stations	1	0	0	6	10	40	10

TABLE 7.—*Summary of the correlation coefficients for the relationship between the 0000 GMT average transport wind speed and the average afternoon surface wind speed.*

Index of correlation	.56–.60	.61–.65	.66–.70	.71–.75	.76–.80	.81–.85	.86–.90	.91–.95
Number of Stations	1	0	10	9	21	12	4	1

TABLE 8.—*Summary of the correlation coefficients for the relationship between average afternoon surface wind speed and the 24-hr. average surface wind speed.*

Index of correlation	.46–.56	.61–.65	.66–.70	.71–.75	.76–.80	.81–.85	.86–.90	.91–.95
Number of stations	3	0	0	3	5	18	35	3

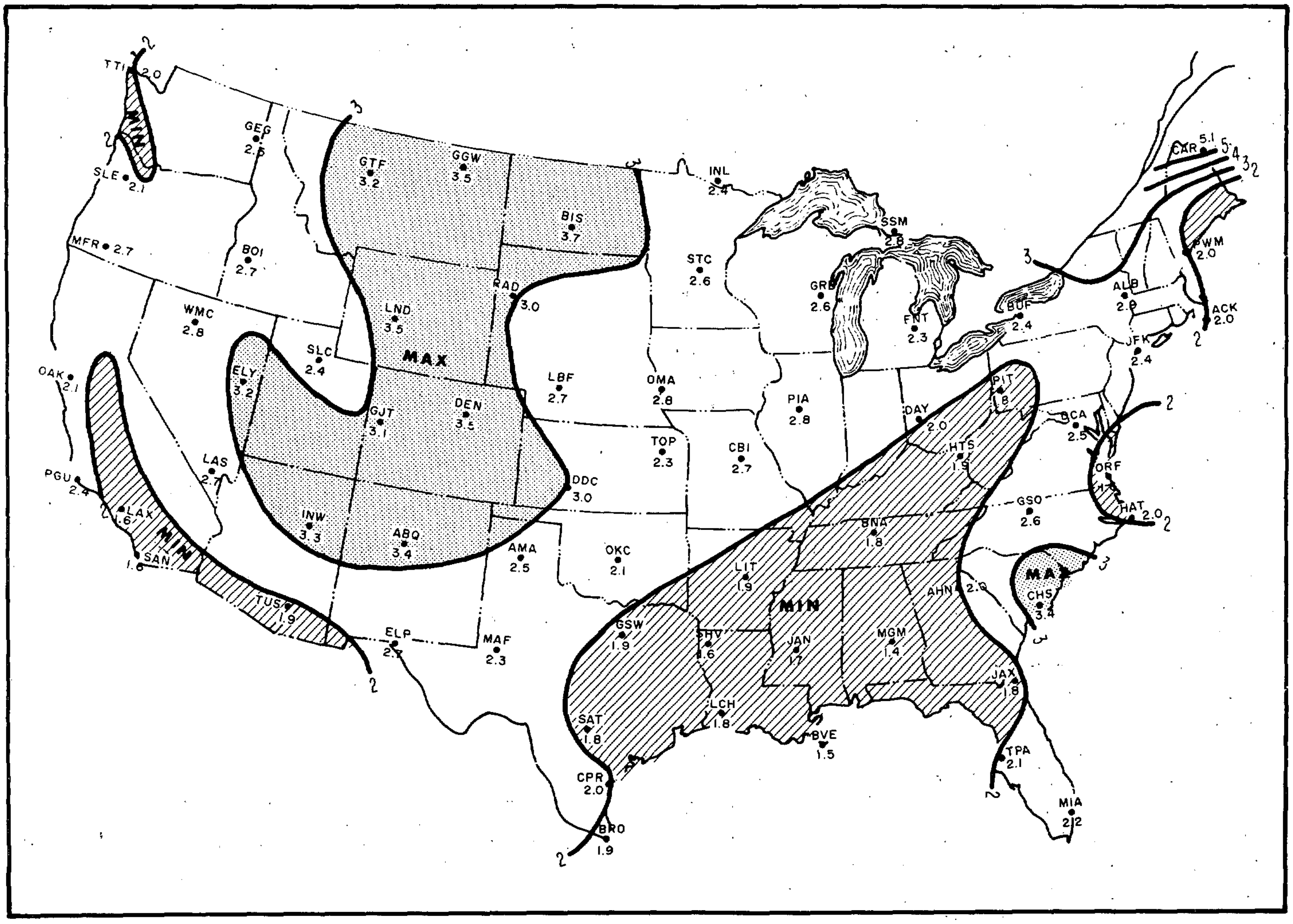

FIGURE 5.—Isopleths of standard errors of transport wind speed (kt.) for the relationship between the transport wind speed and the wind speed nearest the center of the mixing depth.

RELATED WIND SPEEDS

Methods of estimating the afternoon (1200–1600 LST) and 24-hr. average surface wind speeds were also developed in this study. Average afternoon surface wind speeds correlate highly with the 0000 GMT average wind speeds through the mixing depths. In turn, the afternoon wind speeds are useful in estimating 24-hr. average surface wind speeds. Tables 7 and 8 summarize the correlation coefficients about the derived parabolic equations for these relationships.

4. VERIFICATION

A test period was conducted from April 20 through May 9, 1966 to examine results from the methods of forecasting afternoon mixing depths and transport wind speeds. Figure 6 shows an example of such forecasts made from the derived statistical regression equations, NMC forecast material, and forecast surface temperatures. It was assumed that forecast and observed maximum surface temperatures occurred during afternoon hours. Estimates of mixing depth and transport wind speed, derived by use of the above products, were called RADAT forecasts. The RADAT forecasts shown in figure 6 verified on the afternoon of May 5, 1966; they were prepared on the morning (EST) of May 4 when the latest upper-air observations were for 1200 GMT. Figure 7 shows the calculated values on May 5, 1966. Afternoon mixing depths were computed as the height above ground at which the potential temperature of the 24-hr. observed maximum surface temperature intersected the observed 1200 GMT vertical temperature profile; observed transport wind speeds were computed as the 0000 GMT (1900 EST) mean wind speed through the observed afternoon mixing depth. For verification purposes all forecast and calculated values of mixing depths greater than 3 km. were treated as if they were 3-km. depths.

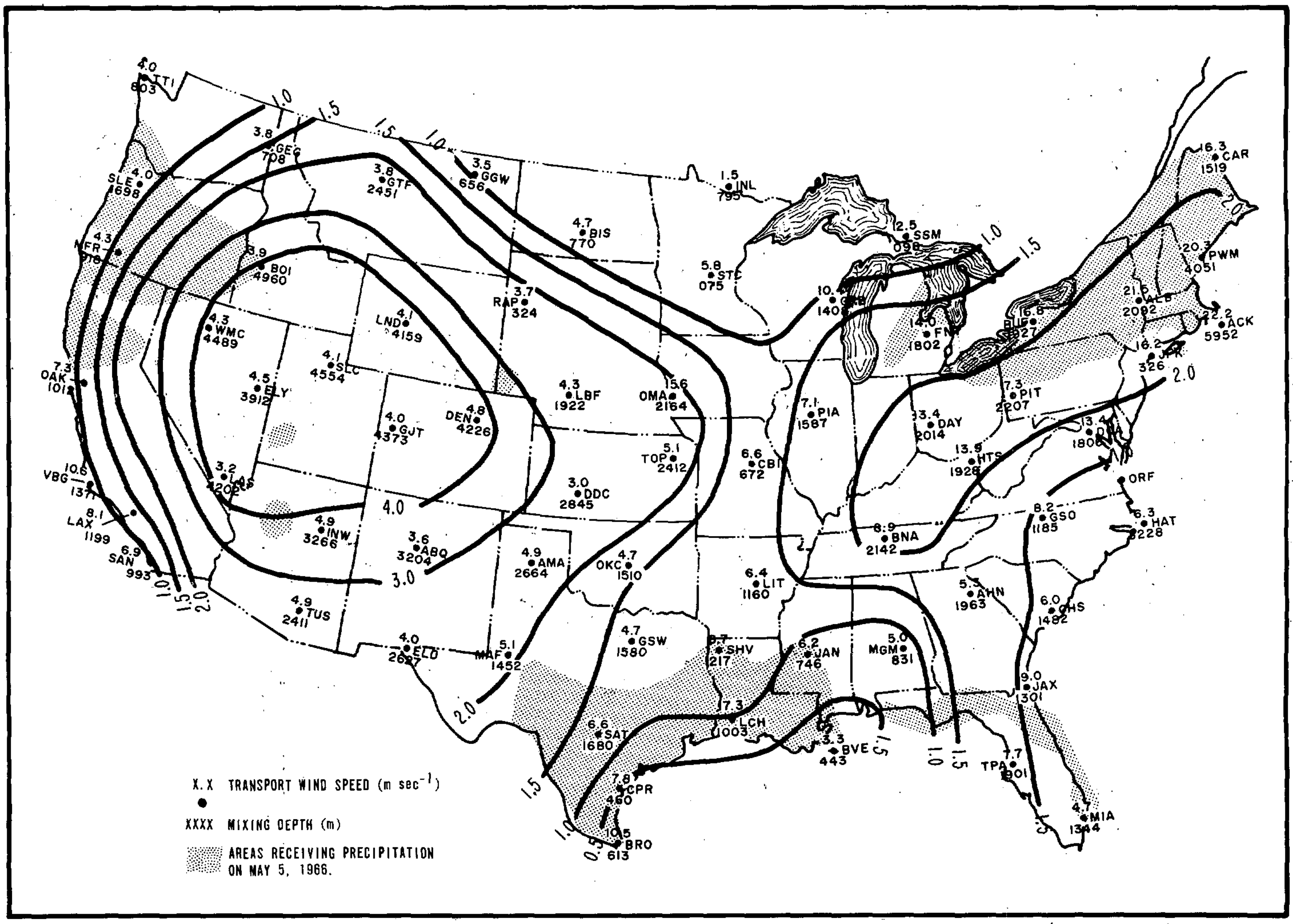

FIGURE 6.—RADAT forecasts of afternoon mixing depth (m.) and transport wind speeds (m. sec.$^{-1}$) made on May 4, 1966. (Isopleths of mixing depth are in kilometers.)

MIXING DEPTHS

Table 9 shows that for the verification test period the observed mean mixing depth was 1790 m.; the mean absolute error of RADAT forecasts for non-precipitation stations was 600 m. and for all stations it was 635 m. These errors may be compared with those for shorter lead time forecasts, i.e., mixing depth forecasts based on today's observed 1200 GMT vertical temperature profiles and forecasts prepared *this morning* of maximum surface temperatures for today. Such forecasts are called RAOB forecasts and, as shown in table 9, the mean absolute errors were only about half those for RADAT forecasts. In the first comparison of table 9, stations where measurable precipitation had occurred during the day were excluded because of the assumption of the dry adiabatic lapse rate in mixing depth calculations.

Mean absolute errors of forecast surface maximum temperatures during the test days are summarized in table 10. Forecast lead times for surface maximum tempera-

TABLE 9.—*Mean absolute errors of afternoon mixing depth from RADAT and RAOB forecasts: April 20 through May 9, 1966*

Observed mean mixing depth	1790 meters	
Mean absolute error of forecast mixing depth	RADAT	RAOB
Excluding stations with precipitation	600 m.	310 m.
Including stations with precipitation	635 m.	360 m.

TABLE 10.—*Mean absolute errors of maximum temperature forecasts: April 20 through May 9, 1966*

	RADAT	RAOB
Excluding stations with precipitation	5.9° F.	3.5° F.
Including stations with precipitation	6.3° F.	4.0° F.

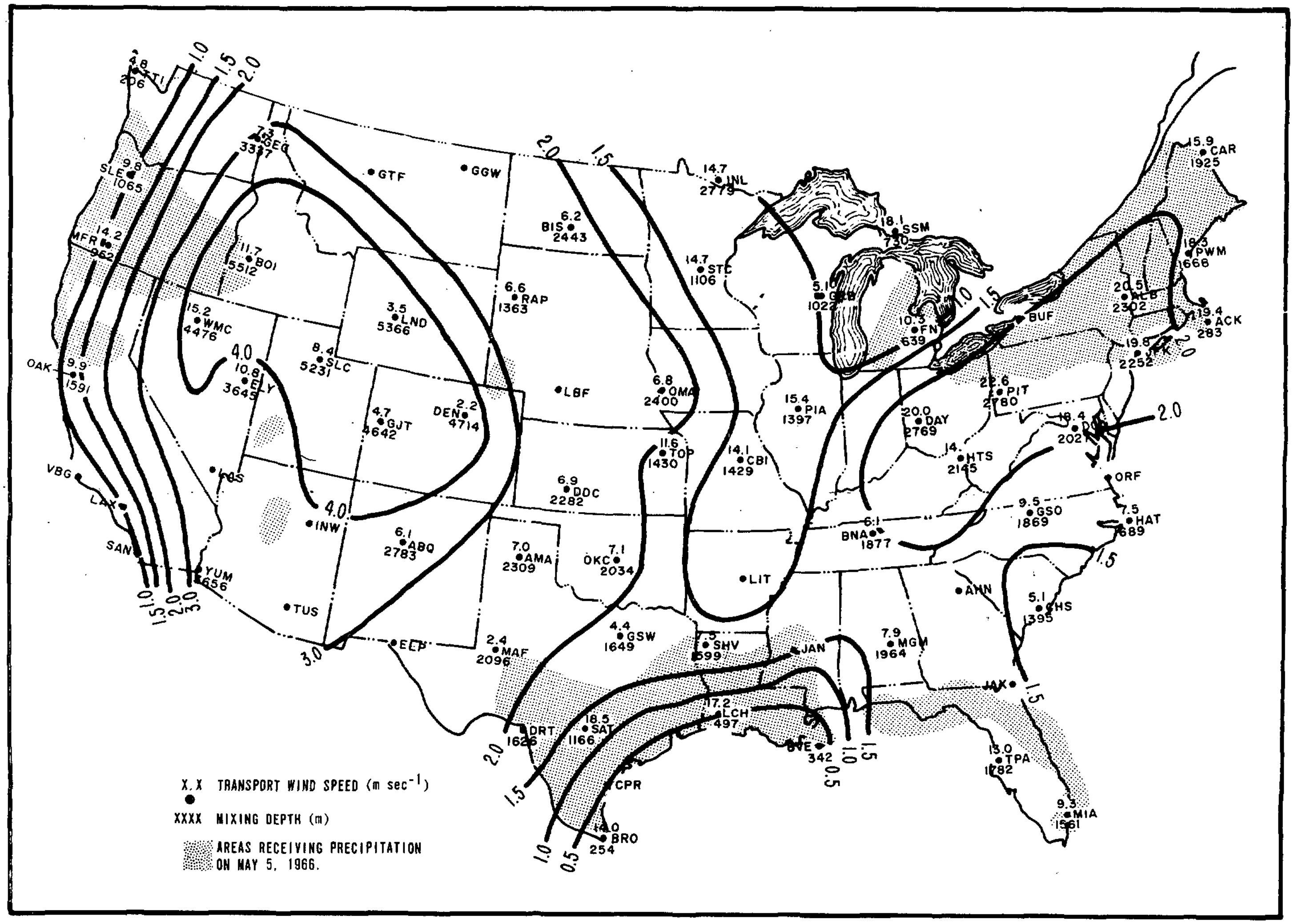

FIGURE 7.—Observed afternoon mixing depths (m.) and transport wind speeds (m. sec.$^{-1}$) on May 5, 1966. (Isopleths of mixing depth are in kilometers.)

tures used in RADAT mixing depth forecasts were 24 hr. longer than those for RAOB mixing depth forecasts. Table 10 shows that errors in forecast temperatures were considerably larger for the longer than for the shorter forecast lead time. The data in tables 9 and 10 suggest that a considerable portion of the error in forecast mixing depths was due to inaccurate forecasts of maximum temperature.

TRANSPORT WIND SPEEDS

Transport wind speed forecasts as calculated from the parabolic regression equations and winds aloft forecasts were verified for only April 21 and May 5. Table 11 summarizes this verification. The 36-hr. RADAT forecasts of average wind speed through the mixing layer utilized NMC wind forecasts with a lead time of 36 hr., i.e., forecasts were based on 1200 GMT upper-air observations yesterday and verified at 0000 GMT tomorrow (1900 EST today). These forecasts of transport wind speed were for mixing depths forecast with a lead time of 30 hr. or more. The 12-hr. RADAT forecasts of average transport wind speed were somewhat better than the 36-hr. RADAT forecasts.

Table 11 also shows that if the 1200 GMT observed average wind speeds through the observed afternoon

TABLE 11.—*Mean absolute errors of transport wind speed from RADAT and persistence forecasts*

	April 21, 1966	May 5, 1966
Observed mean transport wind speed	8.6 m.sec.$^{-1}$	11.1 m.sec.$^{-1}$
Mean absolute error of transport wind speed:		
36-hr. RADAT forecast	3.1	3.7
12-hr. RADAT forecast	2.5	3.0
Mean absolute error of transport wind speed when persistence of the average 1200 GMT wind speed through the *observed* mixing depth is used to indicate the afternoon transport wind speed	3.6	4.5

TABLE 12.—*Comparisons of mixing depth estimates (meters) obtained from 1964 parabolic regression equations for the relationship between mixing depth and ($t_{sfc}-T'$) with estimates for similar equations derived from 1965 data.*

City	Year	$t_{sfc}-T'_{1000\text{-}850}$ (° C.)				$t_{sfc}-T'_{850\text{-}500}$ (° C.)				
		4	6	10	14	18	20	23	26	30
Albany, N.Y.	1964	575	775	1140	1457	1633	1583	1652	1892	2479
	1965	394	687	1131	1388	1484	1550	1715	1958	2406
Bismarck, N. Dak.	1964	411	507	690	859	1142	1189	1467	1993	3078
	1965	429	490	610	728	1118	1269	1606	2075	2905
Columbia, Mo.	1964	550	719	997	1195	1552	1473	1539	1815	2525
	1965	668	743	874	978	1242	1326	1535	1857	2444
Montgomery, Ala.	1964	599	846	1208	1391	1919	1729	1629	1750	2255
	1965	507	784	1241	1565	1589	1502	1519	1711	2241
Oakland, Calif.	1964	590	741	1173	1781	1781	1627	1571	1729	2270
	1965	582	730	1158	1764	1608	1628	1707	1843	2114
Pittsburgh, Pa.	1964	619	726	958	1215	1306	1372	1589	1948	2648
	1965	574	692	1016	1459	1266	1340	1579	1972	2733
Salem, Oreg.	1964	548	704	1087	1565	1447	1483	1634	1904	2448
	1965	586	738	1111	1575	1496	1559	1705	1916	2298
Salt Lake City, Utah	1964	(*)	----	----	----	1215	1642	2529	3711	5746
	1965	----	----	----	----	1241	1662	2429	3357	4847
San Antonio, Tex.	1964	455	692	1059	1283	1497	1450	1540	1822	2498
	1965	681	813	1045	1150	1131	1236	1483	1839	2484
Tucson, Ariz.	1964	(*)	----	----	----	1070	1256	1717	2397	3643
	1965	----	----	----	----	818	1172	1767	2467	3570

*Values not calculated because of the nearness of the station elevation to the 850-mb level.

mixing depths had been used to represent the afternoon transport wind speeds, the mean absolute transport wind speed errors would have been 3.6 m. sec.$^{-1}$ on April 21 and 4.5 m. sec.$^{-1}$ on May 5. Thus 12-hr. persistence forecasts of the observed morning wind speeds would have given poorer estimates of the afternoon transport wind speeds than the 12- and 36-hr. RADAT forecasts, which were based on the derived statistical equations and NMC winds aloft forecasts.

5. SUMMARY

To determine objective forecasts of afternoon mixing depth and the average wind speed through this depth, parabolic regression equations of relationships between these quantities and selected independent variables were derived for 67 rawinsonde stations. Although the methods of obtaining estimates of mixing depths and transport wind speeds were developed specifically for use in the National Air Pollution Potential Forecast Program, the byproducts from this study, statistical equations for estimating average afternoon and 24-hr. surface wind speeds, can be beneficial to synoptic and fire-weather meteorologists.

Methods of forecasting the vertical extent above the surface of atmospheric mixing during the afternoon and the average wind speed through this mixing depth have been presented. Sample forecasts, made during April and May 1966, have shown that the products are satisfactory enough to be incorporated into the National Air Pollution Potential Forecast Program.

6. EPILOGUE

Further justification for using the 1964 derived parabolic regression equations for estimating mixing depths is given in table 12. This table shows, for selected stations, estimates of mixing depths obtained from regression equations derived from 1965 data and similar estimates obtained from the 1964 equations. The values of mixing depths from these equations are not significantly different. Hence, either equations calculated for the two different years or new equations derived from the combination of 1964 and 1965 data can be expected to give similar estimates of mixing depths. At present the 1964 regression equations are used to forecast mixing depths through the year. It is planned to derive parabolic regression equations based on seasonal breakdown of the data. Some improvement in the quality of the forecasts might then be expected.

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REFERENCES

1. J. Badner and M. Kulawiec, "FD Program: Winds Aloft and Temperature Forecasts," *Notes to Forecasters*, Technical Procedures Branch, WXAP Division, U.S. Weather Bureau, Washington, D.C., May 1965.

2. G. Holzworth, "Estimates of Mean Maximum Mixing Depths in the Contiguous United States," *Monthly Weather Review*, vol. 92, No. 5, May 1964, pp. 235–242.
3. M. Miller and L. Niemeyer, "Air Pollution Potential Forecasts—A Year's Experience," *Journal of the Air Pollution Control Association*, vol. 13, No. 5, May 1963, pp. 205–210.
4. B. Ratner, "Upper Air Climatology of the United States," *Technical Paper No. 32*, Part I, U.S. Weather Bureau, June 1957.

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